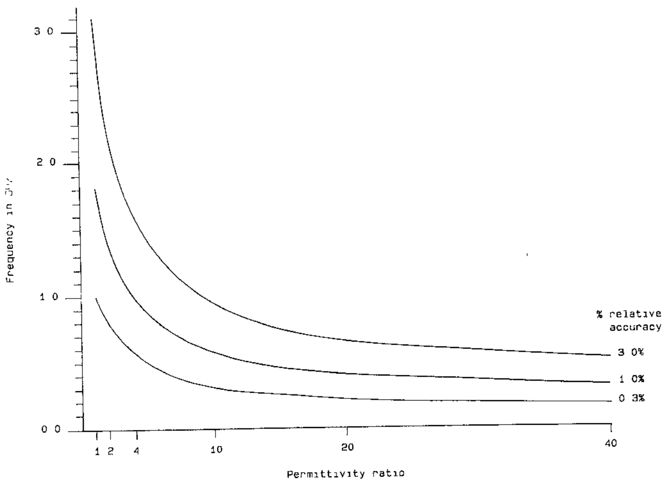

Fig. 2. Relative accuracy bounds for the low-frequency formula.

in the frequency-dependent case with those from the formula. The results of a comparison in the case of the 24.3-Ω line are shown in Fig. 2 and similar results in respect of the 50.0-Ω line of the same outer diameter were also obtained, but they were found to be no less representative than those shown in the figure. For any point on any one of the curves in this figure, there corresponds a permittivity ratio and a frequency beyond which the specified relative accuracy cannot be sustained.

As a matter of interest, the results contained herein were obtained using a desktop micromputer, and all computations involving Bessel functions were carried out using the polynomial approximation representation for such functions [7]. The numbers of modes used to match the field in the annular domain to that in the circular domain were 50 and 100, respectively, and although no attempt has been made to determine the absolute accuracy of the computations, the results compare favourably with those of [2] in the case of the air-filled termination, for which an accuracy of about ±0.1 fF was reported.

Appendix

It may be deduced from an established result in the theory of Fourier–Bessel series that, for the $\lambda_j a$ and $\mu_m a$ as defined in the text, the following formulas are satisfied:

$$\sum_{j=1}^{\infty} \frac{1}{\left(\mu_m^2 a^2 - \lambda_j^2 a^2\right)} \left[\frac{J_0(\lambda_j b)}{J_1(\lambda_j a)}\right]^2 = 0 \tag{A1}$$

and

$$\sum_{j=1}^{\infty} \frac{1}{\left(\mu_m^2 a^2 - \lambda_j^2 a^2\right)^2} \left[\frac{J_0(\lambda_j b)}{J_1(\lambda_j a)}\right]^2 = \frac{1}{4\mu_m^2 a^2}\left[\frac{J_0^2(\mu_m b)}{J_0^2(\mu_m a)} - 1\right]. \tag{A2}$$

Acknowledgment

The author is indebted to Prof. W. E. Williams and Prof. D. Woods of Surrey University.

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A Printed Circuit Stub Tuner for Microwave Integrated Circuits

B. J. MINNIS

***Abstract*—A novel microwave tuning element capable of continuous adjustment has been realized in the form of a planar printed circuit. As such, it is suitable for incorporation into microwave integrated circuits (MIC's), where it can be used for fine-tuning the impedance match between two parts of a circuit when the two parts are either subject to variations due to manufacturing tolerances or are difficult to model. In either case, the tuner is a compact on-circuit tuning facility which does not have to be removed after use. The tuner has been shown to have unique impedance-transforming properties, being capable of matching any realizable impedance to a 50-Ω load. Its main part consists of a coupled-line section and across the gap in the section at different places along its length are positioned two short-circuit bridge conductors. Movement of these bridges produces the variation in impedance transformation.** 3-10/8612437

I. Introduction

Stub tuners in coaxial form have been widely used for many years. Coaxial tuners usually comprise two or three short-circuit stubs of adjustable length connected in shunt with a main "through" line with some appropriate separation. Their chief application has been in the field of microwave measurements where there is a need for continuous adjustment of the impedance presented to a device under test in order to optimize some other parameter such as power transfer or noise figure. Notable examples include load pull measurements on large signal amplifying devices and noise parameter measurements on low-noise FET's.

While being well suited to the measurement role, coaxial tuners or tuners with the same basic circuit concept cannot be included as integral parts of microwave integrated circuits (MIC's). There are certain types of circuit where inclusion of a continuously variable tuning element would be extremely useful. They are mostly narrow-band circuits where perhaps a large spread in device characteristics results in a need for individual circuit tuning or where the use of a nonlinear device has resulted in

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some uncertainty in the circuit design. Frequency multipliers are typical examples.

This paper describes a tuner which is realized as a planar printed circuit and which can therefore form part of a microwave integrated circuit. It is described as a stub tuner because the two-wire equivalent circuit bears a close resemblance to that of the coaxial stub tuner. However, its topology is quite different from that of a coaxial stub tuner and, being sufficiently compact, it can be left in place within a circuit after tuning has been completed. The tuner offers a unique on-circuit tuning/trimming facility which is nondestructive. Any impedance within the unity-radius Smith chart can be matched to 50 Ω.

After a brief description of the tuner, a theoretical treatment will be given, followed by some discussion of physical realization and mode of operation.

II. Tuner Description

An illustration of the tuner is given in Fig. 1, and it can be described in terms of two basic parts. The major part consists of a pair of edge-coupled strip transmission lines across which are positioned a pair of bridge conductors. In the case of a microstrip realization, the bridge conductors are most likely to be bond wires which can be moved at will and fixed in place by an ultrasonic bond, but alternatively a sliding short-circuit bridge guided by a suitable mechanical arrangement can be used. A hand-adjusted insulating probe with a conducting tip is also a possibility. In the case of a triplate stripline realization, the bridge conductors will probably be gold foils which again can be moved at will but would be clamped in position by the two mating surfaces of the triplate, without any need for bonding. As will be illustrated in the theoretical treatment of the circuit, movement of these bridge conductors enables almost any complex terminating impedance to be transformed into and therefore matched to 50 Ω. The most desirable length for the coupled line section has been found to be three quarters of a wavelength at the frequency of operation.

To guarantee a match, the terminating impedance must fall into the plane of the Smith chart (unity radius). There is a small zone of the Smith chart which defines a set of terminating impedances which cannot be matched by the adjustment of the bridge conductors alone, but the zone is small and any impedance contained in the zone can be moved outside by use of the "U" section of line in front of the coupled striplines. As illustrated in Fig. 1, the "U" section of line can be included by reconfiguring relevant bonding wires or strips.

The combination of the edge-coupled strips and the leading "U" section of line makes up the complete tuner and the combination will allow any terminating impedance in the plane of the Smith chart to be matched to 50 Ω. A further attractive feature of the tuner is its ability to pass dc. This allows the tuner to be connected adjacent to an active device without any need for dc breaks.

III. Equivalent Circuit

Although not essential, the tuner will normally use coupled strips of equal width, and this treatment considers only that specific case. Referring to Fig. 2, the coupled strips of the tuner are examined as a cascade of three separate sections. From left to right, the section up to and including the first bridge conductor is equivalent to a line of impedance Z_0 and electrical length θ_1 followed by a shunt, open-circuit stub of the same length and impedance Z_s. The length θ_1 is given by $\theta_1 = (2\pi f\sqrt{\epsilon_r}\, l_1)/c$, where f is frequency, ϵ_r is effective dielectric constant, l_1 is the physical length, and c is the phase velocity in air. Z_0 and Z_s are related to the distributed capacitances of the coupled strips by

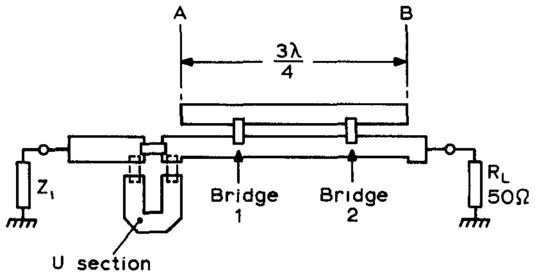

Fig. 1. Tuner schematic diagram.

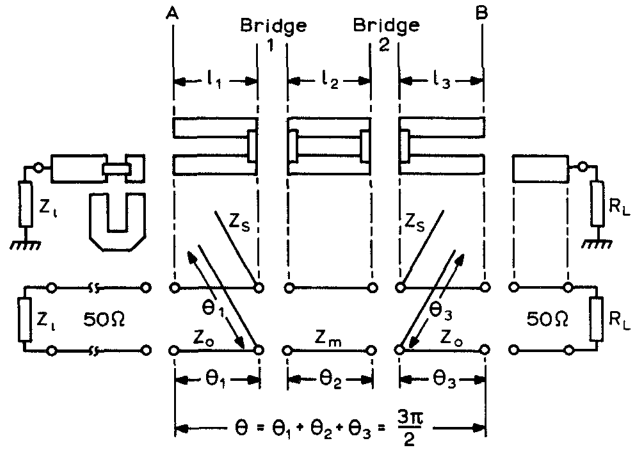

Fig. 2. Tuner equivalent circuit.

$$Z_0 = a\left[\frac{C_a + C_{ab}}{C_a(C_a + 2C_{ab})}\right]$$

and

$$Z_s = a\left[\frac{C_a + C_{ab}}{C_a^2}\right]$$

where a is a constant given by $a = 377/\sqrt{\epsilon_r}$. C_a is the normalized shunt distributed capacitance, and C_{ab} the interstrip capacitance.

The section between the bridge conductors is equivalent to a length of transmission line of impedance Z_m and of electrical length θ_2. Z_m is given by

$$Z_m = \frac{a}{2C_a}.$$

From the second bridge to the end of the strips, the equivalent circuit is the reverse of that of the first section except that the length of the line and stub is θ_3. The electrical lengths θ_2 and θ_3 correspond to the physical lengths l_2 and l_3.

IV. Impedance Transformation

To gain an understanding of the operation of the tuner, it is helpful to examine the variation of input impedance and reflection coefficient at one end of the tuner as the bridge conductors are moved while the opposite end of the tuner is terminated in a matched load. In principle, any combination of values of Z_0, Z_s, and Z_m could be chosen for this, but to give a more meaningful illustration, values have been assigned which correspond to a physically realizable tuner. As will be demonstrated in a later

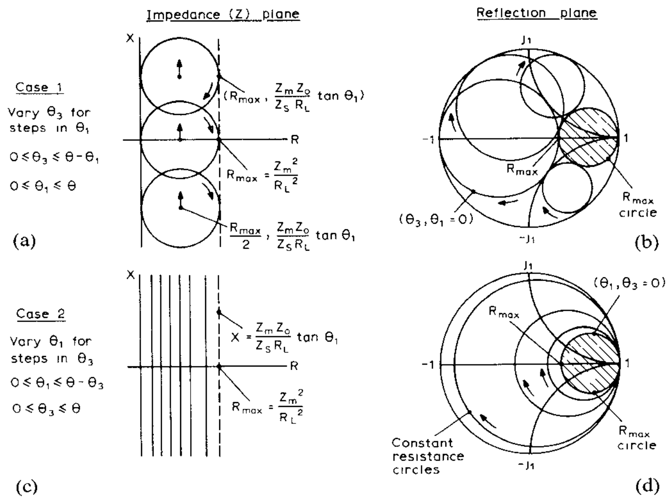

Fig. 3. Tuner input impedance and reflection coefficient versus bridge positions.

section, suitable values are $Z_0 = 114\ \Omega$, $Z_s = 221\ \Omega$ and $Z_m = 75\ \Omega$. Two cases have been treated in this examination, and the corresponding loci of the input impedance and the reflection coefficient have been plotted in Fig. 3. Both the impedance and the reflection coefficient have been normalized to R_l (50 Ω). Case 1 involved adjusting the position of the bridge nearest the matched load for discrete steps of the position of the bridge nearest the input, and case 2 was as case 1 but the adjustment and stepping of the bridges were interchanged. Referring to Fig. 2, case 1 is continuous adjustment of θ_3 for steps in θ_1, while case 2 is a continuous adjustment of θ_1 for steps in θ_3.

As shown in Fig. 3(a), the loci of the input impedance of the tuner as θ_3 is adjusted is a circle of diameter $R_{\max}$. $R_{\max}$ is given by $(Z_m/R_l)^2$. For θ_1 equal to zero, the center of the circle is on the R axis, but as θ_1 is increased, the circle and its center move along a vertical line through $R_{\max}/2$. The corresponding effect in the reflection plane (Smith chart) is shown in Fig. 3(b). The reflection coefficient (ρ) varies as a circle with θ_3, and the circles move in the plane as θ_1 is stepped such that the circles are always tangential to the outer circle of the chart ($\rho = 1$) and another circle, defined as the $R_{\max}$ circle. Shown shaded in Fig. 3(b), the $R_{\max}$ circle represents the area of the Smith chart which is inaccessible with the basic tuner. Fortunately, this area can be small in practice and any impedance falling into this region can be moved outside with a simple phase shift. The leading "U" section of the complete tuner can provide the phase shift.

Fig. 3(c) indicates the loci of input impedance as θ_1 is adjusted. This is a vertical straight line which, for θ_3 equal to zero, crosses the R axis at $R_{\max}$ and which moves between $R_{\max}$ and zero as θ_3 is stepped. Clearly, moving this particular bridge nearest the input of the tuner only affects the imaginary part of the input impedance, and this is a most interesting and unusual property. The corresponding loci in the reflection plane are constant-resistance circles, all of which are tangential to the outermost circle at the point $\rho = +1$. $R_{\max}$ defines the smallest of these circles.

V. Choice of Electrical Length

It is necessary for the electrical length of the coupled-line section of the tuner to be greater than π radians (i.e., a half-wavelength at the operating frequency). This is so that the shunt stubs in the equivalent circuit will be capable of providing the full range of susceptance ($-\infty$ to $+\infty$) in shunt with the main through path. In addition, the length must not be π or a multiple or π radians since in such circumstances all the circles corresponding to parts (b) and (d) of Fig. 3 are coincident and the tuner is effectively useless. Apart from these two basic conditions, choosing length is a matter of achieving the maximum tuning range with a minimum of sensitivity to bridge position.

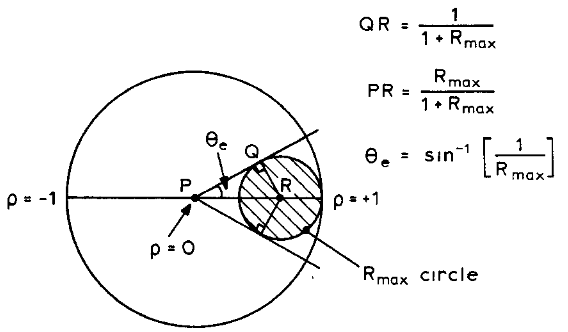

Fig. 4. Electrical length calculation for 'U' section.

With these objectives, a choice of three quarters of a wavelength turns out to be the most desirable length. It is true that as the length of the tuner is reduced towards a half-wavelength, there is an increase in $R_{\max}$ and therefore an effective increase in tuning range, but this advantage is more than offset by the considerable increase in tuning sensitivity. There is no advantage to be gained by making the length a larger multiple of a quarter-wavelength.

Choosing the length of the leading "U" section of the tuner is a matter of working out how much phase shift is required to move an impedance from one edge of the $R_{\max}$ circle to the other. Referring to the diagram in Fig. 4, the relevant angle is θ_e, and after considering the radius and coordinates of the center of the $R_{\max}$ circle, θ_e is given by $\sin^{-1}(1/R_{\max})$. For the figures used earlier, the minimum value of θ_e is calculated to be 26.4°.

VI. Formulas

The behavior of the tuner as described above has been chiefly determined by numerical methods. This involved using a suitably modified CAD package capable of analyzing the equivalent circuit of the tuner at a particular frequency while sweeping the values of l_1 and l_3 between appropriate limits. It was more convenient to perform the analysis in this way rather than derive analytical expressions for input impedance in terms of l_1 and l_3 because although deriving such expressions is not difficult, finding a form for the expressions which is recognizable as that of a specific function such as a circle is indeed difficult without prior knowledge. Having gained prior knowledge, however, the following formulas were derived for the real and imaginary parts of the input impedance of the tuner as a function of the positions of the two bridges. They relate specifically to the case of a three-quarter-wavelength tuner, and when normalized to R_l, they can be seen to be consistent with the diagrams shown in parts (a) and (c) of Fig. 3. In the formulas, bridge positions have been translated into electrical lengths θ_1 and θ_3:

$$\mathrm{Re}(Z_{\mathrm{in}}) = \frac{Z_m^2}{R_l^2 + (Z_0 Z_m Y_s \tan\theta_3)^2} \times R_l$$

and

$$\mathrm{Im}(Z_{\mathrm{in}}) = Z_0 Z_m Y_s \left[\tan\theta_1 - \frac{\mathrm{Re}(Z_{\mathrm{in}})\tan\theta_3}{R_l}\right]$$

where $Y_s = 1/Z_s$.

VII. Physical Realization

To maximize the tuning capability of the coupled section of the tuner, it has been shown that the impedance of the central section (Z_m in the equivalent circuit) must be made as high as possible. This will be achieved by making the width and the separation of the coupled lines as small as possible, irrespective of a realization in microstrip or stripline. The relationship between physical dimensions and distributed capacitances has been widely reported for both microstrip and stripline (e.g., [1] and [2]), and by choosing line dimensions and separations which are small but readily realizable, this information can be used to calculate the element values in the corresponding equivalent circuit. Use is often made in the literature of the concept of odd- and even-mode impedances (Z_{0o} and Z_{0e}), which, in terms of distributed capacitances, are given by

$$Z_{0o} = a/(Ca + 2Cab)$$

and

$$Z_{0e} = a/Ca.$$

A stripline realization has been used as a practical example. Choosing Z_m to be 75 Ω gives Z_{0e} a value of 150 Ω and from experience, a value for C_{ab} of 0.8 is usually realizable. If a dielectric constant of 2.2 is assumed, the above formulas give Z_{0o} a value of 77.15 Ω. Corresponding values of Z_0 and Z_s are 114 and 221 Ω. Reference to the relevant literature confirms that this coupled section will be realizable.

The line width of the leading "U" section of the tuner is chosen for a characteristic impedance of 50 Ω.

VIII. Using the Tuner

The object in adjusting the bridge conductors will usually be to achieve an optimum match between two impedances. However, the parameter used to indicate the quality of the match will vary with the specific application. It could simply be the reflection coefficient but could alternatively be gain or power output in the case of an amplifier, for example. This description will refer to the use of reflection coefficient and the problem of matching a complex impedance to a 50-Ω load.

Referring to parts (b) and (d) of Fig. 3, it is apparent that adjustment of either bridge can always move a given impedance from a position outside the R_{max} circle towards the center of the Smith chart. Which bridge produces the most pronounced effect will depend upon the location of the relevant impedance in the chart. For impedances near $\rho = +1$, changes in θ_1 (referring to Fig. 2) will be most effective, but changing θ_3 will be most effective for impedances near $\rho = -1$. An interactive procedure, therefore, consisting of successive adjustments of θ_1 and θ_3 for minimum reflection coefficient will result in a match condition when the complex impedance falls outside the R_{max} circle. If a match condition cannot be obtained, then it should be assumed that the impedance falls within the R_{max} circle and the leading "U" section of the tuner must be brought into play before making a second attempt.

Since adjustment of either θ_1 or θ_3 should bring about an improvement in match, the logical starting positions for the bridge conductors are the two respective ends of the section.

IX. Conclusions

A printed circuit stub tuner has been described which is continuously adjustable and capable of matching any complex impedance in the unit radius reflection plane to 50 Ω. With its simple planar construction, it is particularly useful in microwave integrated circuits where variations in the parameters of an active device call for on-circuit tuning.

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Phase Noise Reduction in FET Oscillators by Low-Frequency Loading and Feedback Circuitry Optimization

M. PRIGENT AND J. OBREGON

***Abstract*—Optimization of low-frequency loading allows reduction of 1/f converted noise in FET oscillators. Moreover, an appropriate low-frequency feedback between drain and gate gives an improvement over previous results. This improvement is explained by taking into account the properties of the noise autocorrelation function.**

I. Introduction

Several ways have been proposed to reduce $1/f$ noise conversion in transistor oscillators [1]–[3]. In this paper, we present an analysis of the low-frequency loading and feedback effects on $1/f$ noise up conversion which enables us to experiment successfully with a circuit for reducing FM noise near the carrier. This carefully designed low-frequency circuit makes it possible to improve the oscillator noise characteristics without any variations of the bias voltages, oscillation frequency, or output power. The only variation observed was the FM noise, as indicated by the noise measurements.

II. Explanation of $1/f$ Noise Conversion Improvement Due to Low-Frequency Circuitry

A. Optimization of Low-Frequency Loading

In a first approximation, $1/f$ FET noise may be modeled by a gate voltage noise generator. When the FET oscillates, the EMF of this equivalent noise generator depends on the microwave amplitude. However, for a given oscillating amplitude, the EMF of the $1/f$ equivalent noise generator has a fixed value and may be extracted. Therefore, from the low-frequency point of view, a noisy FET placed in an oscillating circuit may be approximated by a gate voltage noise generator ($1/f$ noise) in series with a noise-free nonlinear two-port. The input and output currents, I_g and I_d, are both functions of input and output voltages V_{gs} and V_{ds}, as indicated in Fig. 1 [4].

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